PHYSICAL REVIEW D **71,** 053003 (2005)

Lepton flavor violation in extra dimension models

We-Fu Chang[1,2,*] and John N. Ng[2,†]
[1]*Institute of Physics, Academia Sinica, Taipei 115, Taiwan*
[2]*TRIUMF Theory Group, 4004 Wesbrook Mall, Vancouver, B.C. V6T 2A3, Canada*
(Received 18 January 2005; published 10 March 2005)

Models involving large extra spatial dimension(s) have interesting predictions on lepton flavor violating processes. We consider some five-dimensional (5D) models which are related to neutrino mass generation or address the fermion masses hierarchy problem. We study the signatures in low energy experiments that can discriminate the different models. The focus is on muon-electron conversion in nuclei $\mu \to e\gamma$ and $\mu \to 3e$ processes and their τ counterparts. Their links with the active neutrino mass matrix are investigated. We show that in the models we discussed the branching ratio of $\mu \to e\gamma$ like rare process is much smaller than the ones of $\mu \to 3e$ like processes. This is in sharp contrast to most of the traditional wisdom based on four-dimensional (4D) gauge models. Moreover, some rare tau decays are more promising than the rare muon decays.

DOI: 10.1103/PhysRevD.71.053003 PACS numbers: 13.35.Bv, 11.10.Kk, 13.35.Dx

I. INTRODUCTION

In the standard model (SM) with 15 fermions per family neutrinos are strictly massless and the charged leptons' weak eigenstates can be chosen to be their mass eigenstates. Thus, each generation has a separately conserved lepton number. If one neglects the tiny effects from nonperturbative processes, there is no lepton flavor violating (LFV) interaction in SM. However, recent neutrino experiments show strong evidence that neutrinos have no zero masses and the three active neutrinos mix [1–4]. Most physicists take this to be a harbinger of new physics beyond the SM. Moreover, finite neutrino masses alone would imply the existence of LFV in charged lepton sector analogous to the quarks. If so we expect the Glashow-Iliopoulos-Maiani (GIM) mechanism to be operative in the lepton sector then the rate of induced LFV processes will be proportional to the neutrino mass square difference, which is of the order of $<10^{-3}(\mathrm{eV})^2$. Hence, they will be hopelessly small for experimental verification. Therefore, additional ingredients are essential for a detectable LFV signature. It is very common in model building to have the new physics that generate neutrino masses also give rise to LFV reactions. This link appears to be natural although there is no guarantee that this is the case in nature. With this cautionary note we will focus attention to new physics that links the two phenomena.

Among the numerous beyond the SM models, LFV signatures are most intensely studied in supersymmetric (SUSY) ones. The connection with neutrino masses is established through the seesaw mechanism which is the orthodox way of getting a small mass for the active neutrinos. Since the latter has a natural setting in grand unified theories (GUT) the end results are rather bedecked supersymmetric seesaw models; see, e.g., [5]. Although the details are different the generic source of LFV lies in the mixing of various sfermions. The right-handed Majorana neutrinos play a secondary role in this class of models. In general it is natural to expect $B(\mu \to e\gamma) \gg B(\mu \to 3e)$ in SUSY models.

For nonsupersymmetric models neutrino mass generation via the seesaw mechanism would require the right-handed neutrinos to be of the GUT scale. In this simplest version all LFV are undetectable. Attempts are now made to lower some right-handed neutrinos mandated by the seesaw mechanism to the TeV scale so that the seesaw mechanism itself can be tested experimentally. If so then one can optimistically anticipate LFV signatures in the next round of experiments [6]. Independent of the details of the models one again expects $B(\mu \to e\gamma) \gg B(\mu \to 3e)$ to hold true.

Recently a new avenue has opened up in the construction of models beyond the SM that exploits the possible existence of extra spatial dimensions. These theories are particularly interesting phenomenologically in the brane world context. It is fascinating that many long standing problems in the usual four-dimensional (4D) field theories can be overcome or take on new perspectives in these higher dimensional constructs. For example the hierarchy problem is solved by invoking large extra dimensions. In this note, we would like to draw the readers' attention to the models which involve one or more flat extra spatial dimensions. Furthermore, we focus on those that address the neutrino mass problem. In some cases, we predict a reversed pattern of $B(\mu \to 3e) \gg B(\mu \to e\gamma)$ compared to SUSY models. On the experimental side, it shall be interesting to see this.

The current experimental limits on muon LFV have already put very stringent constraints on model building. On the other hand, the limits from tau LFV are rather loose. We shall constrain the extra dimension models by the muon rare processes data and place upper limits on the rare decays of the τ. To avoid any hadronic uncertainties

*Electronic address: wfchang@phys.sinica.edu.tw
†Electronic address: misery@triumf.ca

we shall focus on purely leptonic processes. We shall also discuss the possible ways to discriminate different models and their connections to neutrino masses.

In this brief review, we give few examples of extra dimension models which give potentially testable LFV signatures. These LFV processes are all directly or indirectly related to the generation of neutrino masses. We compare the LFV processes in a five dimension (5D) $SU(3)_W$ [7] and SU(5) [8] GUT models where neutrino Majorana masses are generated radiatively without a right-handed neutrino which is a viable but less discussed alternative to the seesaw mechanism. A brief review of this construction is given in [9]. Also, we discuss the LFV processes in split fermion or multibrane scenario.

The following is our plan for the paper. In Sec. II, we will first review the general operator analysis for the lepton flavor violating processes. This will also set the notations for the rest of the discussions. Section III examines LFV in a 5D $SU(3)_w$ model. New results of the one-loop calculations are given here. For details of the model and neutrino mass generation we refer to [7]. In Sec. IV, the LFV processes induced by 5D SU(5) model will be discussed. The discussion here has not been presented before. The alternative way of studying the flavor problem using the split fermion model is examined in Sec. V. Calculations of the LFV processes in this scenario involves many new unknown parameters. The models lack predictive power even semiquantitatively. However, very general generic trends for LFV can be discerned even in this early stage of development. Our conclusions are given in Sec. VI. The necessary 5D gauge fixing details, which is crucial for loop calculations, are presented in Appendix A and B.

II. GENERAL OPERATOR ANALYSIS

First of all, we collect the necessary general formulas for the study of LFV processes. The most important ones are the effective interactions of $L - l - \gamma$ and $L - l - Z$ where we use the notation L to denote the heavier charged lepton which usually is either μ or τ and l is the lighter daughter lepton which can be μ or e.

In LFV studies, the most important contribution comes from the effective $L - l - \gamma$ vertex. The similar vertex where a virtual Z replaces the γ is subdominant in the class of models we are considering. For definiteness we will take $L = \mu$ and $l = e$. The most general $\mu - e - \gamma$ interaction amplitude allowed by Lorentz and gauge invariance can be written as:

$$\mathcal{M} = -eA^*_\mu(q)\overline{u}_e(p_\mu - q)\Big\{[f_{E0}(q^2) + f_{M0}(q^2)\gamma^5] \times \gamma_\nu\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) + [f_{M1}(q^2) + f_{E1}(q^2)\gamma^5] \times \frac{i\sigma^{\mu\nu}q_\nu}{m_\mu}\Big\}u_\mu(p_\mu) \quad (1)$$

with the convention $e = |e| > 0$ used throughout this paper and q^μ is the photon 4-momentum. For real photon emission, only f_{E1} and f_{M1} contribute. But if an off-shell photon is involved, then all four form factors contribute. After proper renormalization, the amplitude is finite as $q^2 \to 0$, so we must have $f_{E0}(0) = f_{M0}(0) = 0$. It is customary to factor out q^2 and rewrite the electric and magnetic form factors as

$$f_{E0}(q^2) = \frac{q^2}{m_\mu^2}\tilde{f}_{E0}(q^2), \qquad f_{M0}(q^2) = \frac{q^2}{m_\mu^2}\tilde{f}_{M0}(q^2), \quad (2)$$

and now $\tilde{f}_{E0}(q^2)$ and $\tilde{f}_{M0}(q^2)$ are finite at $q^2 \to 0$.

A. $L \to l_1 l_2 \bar{l}_3$ and $L \to l\gamma$

Using similar notations of [10], the most general effective Lagrangian for $\mu \to 3e$ and $\mu \to e\gamma$ can be expressed as:

$$\begin{aligned}-\frac{\sqrt{2}\mathcal{L}}{4G_F} = {} & m_\mu A_R \overline{e}_R \sigma^{\mu\nu}\mu_L F_{\mu\nu} + m_\mu A_L \overline{e}_L \sigma^{\mu\nu}\mu_R F_{\mu\nu} \\ & + g_1(\overline{e}_R\mu_L)(\overline{e}_R e_L) + g_2(\overline{e}_L\mu_R)(\overline{e}_L e_R) \\ & + g_3(\overline{e}_R\gamma^\mu\mu_R)(\overline{e}_R\gamma_\mu e_R) + g_4(\overline{e}_L\gamma^\mu\mu_L) \\ & \times (\overline{e}_L\gamma_\mu e_L) + g_5(\overline{e}_R\gamma^\mu\mu_R)(\overline{e}_L\gamma_\mu e_L) \\ & + g_6(\overline{e}_L\gamma^\mu\mu_L)(\overline{e}_R\gamma_\mu e_R) + \text{H.c.},\end{aligned} \quad (3)$$

where

$$\begin{aligned}A_R &= -\frac{\sqrt{2}e}{8G_F m_\mu^2}[f_{E1}(0) + f_{M1}(0)], \\ A_L &= -\frac{\sqrt{2}e}{8G_F m_\mu^2}[f_{M1}(0) - f_{E1}(0)].\end{aligned} \quad (4)$$

Also note the anapole form factors f_{E0} and f_{M0} have vector like effective contributions to g_{3-6}:

$$\delta g_3 = \delta g_5 = \frac{\sqrt{2}e^2}{4G_F m_\mu^2}[\tilde{f}_{E0}(0) - \tilde{f}_{M0}(0)], \quad (5)$$

$$\delta g_4 = \delta g_6 = \frac{\sqrt{2}e^2}{4G_F m_\mu^2}[\tilde{f}_{E0}(0) + \tilde{f}_{M0}(0)], \quad (6)$$

which shall be included in the $g_{3,4,5,6}$. The above effective Lagrangian leads to

$$B(\mu \to e\gamma) = 384\pi^2(|A_L|^2 + |A_R|^2), \quad (7)$$

$$\begin{aligned}B(\mu \to 3e) = {} & \frac{|g_1|^2 + |g_2|^2}{8} + 2(|g_3|^2 + |g_4|^2) \\ & + |g_5|^2 + |g_6|^2 + 8eRe[A_R(2g_4^* + g_6^*) \\ & + A_L(2g_3^* + g_5^*)] + 64e^2\left\{\ln\frac{m_\mu}{m_e} - \frac{11}{8}\right\} \\ & \times (|A_R|^2 + |A_L|^2)\end{aligned} \quad (8)$$

if electron mass is ignored.

To carry out the calculation, it is convenient to define two dimensionless variables $x_1 = 2E_1/m_\mu$ and $x_2 = 2E_2/m_\mu$. However, it is important to keep m_e^2 terms in the intermediate steps in order to properly extract the finite term in the last line of Eq. (8). Our result agrees with [10,11].

The expressions of Eq. (8), except the last line of dipole operators, can also apply to $\tau \to l + \gamma$ and $\tau \to l_1 l_1 \bar{l}_3$ processes . For $\tau \to ee\bar{\mu}, \mu\mu\bar{e}$ processes, the part from dipole operators have double loop suppression from two flavor violation vertices and resulting in an insignificant branching ratios thus can be safely ignored. For τ decay, the branching ratios given above are normalized to $B(\tau \to e\bar{\nu}_e\nu_\tau)$. This holds for subsequent discussions of τ decays.

To complete the story, we also give the expression for processes with no identical particles in the final state, namely, $\tau \to \mu\bar{e}e, e\mu\bar{\mu}$, or $l_1 \neq l_2 = l_3$. The above expression for the branching ratio is now modified to:

$$\begin{aligned} B(\tau\to l_1 l_2 \bar{l}_3) = {} & \frac{|g_1|^2+|g_2|^2}{4} + (|g_3|^2+|g_4|^2+|g_5|^2+|g_6|^2) \\ & + 8eRe[A_R(g_4^*+g_6^*)+A_L(g_3^*+g_5^*)] \\ & + 64e^2\left\{\ln\frac{m_\tau}{m_2}-\frac{3}{2}\right\}(|A_R|^2+|A_L|^2) \end{aligned} \tag{9}$$

with trivial extension of g_is. In arriving the last line of Eq. (9), we have ignored the masses difference between m_e and m_μ in phase space integration but keep the crucial mass singularity associated with the virtual photon. Not surprisingly, the approximation agrees very well with the actual numerical integrations.

If the photonic dipole operator is the only dominate LFV source, we have the following model-independent prediction for

$$B(\tau \to e\gamma) = B(\tau \to \mu\gamma), \tag{10}$$

$$\frac{B(\tau \to \mu e\bar{e})}{B(\tau \to e\gamma)} = \frac{2\alpha}{3\pi}\left\{\ln\frac{m_\tau}{m_e} - \frac{3}{2}\right\}, \tag{11}$$

$$\frac{B(\tau \to e\mu\bar{\mu})}{B(\tau \to e\gamma)} = \frac{2\alpha}{3\pi}\left\{\ln\frac{m_\tau}{m_\mu} - \frac{3}{2}\right\}, \tag{12}$$

to the accuracy of m_μ^2/m_τ^2. To our knowledge, the last two relations have not been presented before.

B. $\mu - e$ conversion in nuclei

We can write the effective LFV Lagrangian for $\mu - e$ conversion as:

$$\begin{aligned} \frac{\mathcal{L}_{\rm eff}}{\sqrt{2}G_F} = {} & \bar{e}(s - p\gamma^5)\mu\sum_q \bar{q}(s_q - p_q\gamma^5)q \\ & + \bar{e}\gamma^\alpha(v - a\gamma^5)\mu\sum_q \bar{q}\,\gamma_\alpha(v_q - a_q\gamma^5)q \\ & + \frac{1}{2}\bar{e}(t_s + t_p\gamma^5)\sigma^{\alpha\beta}\mu\sum_q \bar{q}\,\sigma_{\alpha\beta}q + \text{H.c.} \end{aligned} \tag{13}$$

with self-explanatory notations. Here, flavor changing terms in the quark sector are not included since they are not expected to be important here. The effective couplings are normalized to $(\sqrt{2}G_F)^{1/2}$. For example, the SM Z boson has a vector coupling to quarks given by

$$v^q = T_3 - 2Q\sin^2\theta.$$

To calculate the conversion rate, we need to promote the interaction from quark level to the nucleon level by computing the matrix elements $\langle N|\bar{q}\Gamma q|N\rangle = G_\Gamma^{q,N}\bar{N}\Gamma N$ where N denotes a nucleon and $\Gamma = \{1, \gamma^5, \gamma_\alpha, \gamma_\alpha\gamma^5, \sigma_{\alpha\beta}\}$. Since the coherent process is the important one only vector and scalar operators matter:

$$\langle p|\bar{q}\gamma_\alpha q|p\rangle = G_V^{q,p}\bar{p}\gamma_\alpha p, \qquad \langle n|\bar{q}\gamma_\alpha q|n\rangle = G_V^{q,n}\bar{n}\gamma_\alpha n, \tag{14}$$

and

$$\langle p|\bar{q}q|p\rangle = G_S^{q,p}\bar{p}p, \qquad \langle n|\bar{q}q|n\rangle = G_S^{q,n}\bar{n}n. \tag{15}$$

By conserving of vector current, in the $q^2 \sim 0$ limit, one can determine that $G_V^{u,p} = G_V^{d,n} = 2$ and $G_V^{u,n} = G_V^{d,p} = 1$. However, one has to rely on the nucleon model to evaluate the scalar operator. For qualitative estimation, we will use the result $G_S \sim G_V$ from full nonrelativistic quark model but the reader should keep in mind that the uncertainty of nucleon model could be as large as few tens percent [12]. Following the approximations used in [13], the conversion rate, normalized to the normal muon capture rate $\Gamma_{\rm capt}$, can be expressed as [10,13,14]:

$$\begin{aligned} B_{\rm conv} = {} & \frac{p_e E_e G_F^2 F_p^2 m_\mu^3 \alpha^3 Z_{\rm eff}^4}{2\pi^2 Z\Gamma_{\rm capt}}\{|4eA_L Z + (s-p)S_N \\ & + (v-a)Q_N|^2 + |4eA_R Z + (s+p)S_N \\ & + (v+a)Q_N|^2\} \end{aligned} \tag{16}$$

by assuming that the proton and neutron density are equal and the muon wave function does not change very much in the nucleus, and F_p is a form factor whose definition can be found in [13] and $p_e(E_e)$ is the electron momentum (energy), $E_e \sim p_e \sim m_\mu$. For ${}^{48}_{22}Ti({}^{27}_{13}Al)$, $F_p \sim 0.55(0.66)$, $Z_{\rm eff} \sim 17.61(11.62)$, and $\Gamma_{\rm capture} \sim 2.6(0.71) \times 10^6 s^{-1}$ [15].

Where the coherent vector and scalar coupling strength of nuclei N are defined as

$$S_N \equiv s^u(2Z + N) + s^d(2N + Z), \tag{17}$$

$$Q_N \equiv v^u(2Z + N) + v^d(2N + Z). \tag{18}$$

If there are more than one gauge or scalar bosons mediating this process, the above expression can be trivially extended with modified couplings:

$$(s \pm p)S_N \Rightarrow \sum_i (s^i \pm p^i)S_N^i \frac{M_Z^2}{M_{Hi}^2}, \tag{19}$$

$$(v \pm a)Q_N \Rightarrow \sum_i (v^i \pm a^i)Q_N^i \frac{M_Z^2}{M_{Zi}^2}. \tag{20}$$

Note that the form factors $\tilde{f}_{E0}$ and $\tilde{f}_{M0}$ in Eq. (2) have extra contribution to the vector couplings:

$$\delta v = -\frac{2eM_W}{gm_\mu}\tilde{f}_{E0}, \qquad \delta a = -\frac{2eM_W}{gm_\mu}\tilde{f}_{M0}, \qquad \delta v_q = \frac{2eM_W}{gm_\mu}Q_q, \tag{21}$$

and if Eq. (2) is the only LFV source, then Eq. (16) reduces to the well-known formula given in [13]

$$B^\gamma_{\rm conv} = \frac{8m_\mu F_p^2 \alpha^5 Z_{\rm eff}^4 Z}{\Gamma_{\rm capt}}\{|f_{M1} + f_{E0}|^2 + |f_{M0} + f_{E1}|^2\}. \tag{22}$$

Also a model-independent relation between the $\mu - e$ conversion and the $\mu \to e\gamma$

$$B^\gamma_{\rm conv} = \frac{m_\mu^5 G_F^2 F_p^2 \alpha^4 Z_{\rm eff}^4 Z}{12\pi^3 \Gamma_{\rm capt}} \times \left(\frac{|f_{M1} + f_{E0}|^2 + |f_{M0} + f_{E1}|^2}{|f_{M1}|^2 + |f_{E1}|^2}\right) B(\mu \to e + \gamma). \tag{23}$$

The above brief review is sufficient for the phenomenological analysis we do. Next, we will head for the extra-dimensional models and discuss their LFV signatures.

III. 5D $SU(3)_W$ UNIFICATION MODEL

It has been known for a long time that the SM lepton left-handed doublet and the right-handed singlet charged lepton in each family can beautifully form an $SU(3)_W$ fundamental representation, i.e., $L = (e, \nu, e^c)_L^T$ [16]. This is implemented in an electroweak only unification in which $SU(2) \times U(1)$ is unified to $SU(3)_W$. One of the attractive points of this unification model is the tree-level prediction of $\sin^2\theta_W = 1/4$. Renormalization group considerations point to a relatively low scale of unification at $\sim$ few TeV. We shall use $\{U^{\pm 2}, V^{\pm}\}$ to denote the $SU(3)_W/[SU(2) \times U(1)]$ gauge bosons which have SM quantum number $(2, \pm 3/2)$. In 4D, the $SU(3)_W$ GUT has a fundamental difficulty of embedding quarks into $SU(3)_W$ representations. This problem can be circumvented by promoting the model into five-dimensional space time [7,17]. We give a brief summary of the model construction here.

The extra spatial dimension, with coordinate denoted by y, is compactified into an $S_1/(Z_2 \times Z_2')$ orbifold. Where the circle S_1 of radius R, or $y = [-\pi R, \pi R]$, is orbifolded by a Z_2 which identifies points y and $-y$. The resulting space is further divided by a second Z_2' acting on $y' = y - \pi R/2$ to give the final geometry.

We now have two parity transformations $P: y \leftrightarrow -y$ and $P': y' \leftrightarrow -y'$ under which the bulk fields can be assigned either of the eigenvalues $+$ or $-$. This freedom is used to break the bulk $SU(3)_W$ symmetry to $SU(2) \times U(1)$. Explicitly, one assigns the following properties to bulk gauge fields

$$\begin{aligned} \mathcal{A}_\mu(y) &= \mathrm{P}\mathcal{A}_\mu(-y)\mathrm{P}^{-1}, \\ \mathcal{A}_\mu(y') &= \mathrm{P}'\mathcal{A}_\mu(-y')\mathrm{P}'^{-1}, \\ \mathcal{A}_5(y) &= -\mathrm{P}\mathcal{A}_5(-y)\mathrm{P}^{-1}, \\ \mathcal{A}_5(y') &= -\mathrm{P}'\mathcal{A}_5(-y')\mathrm{P}'^{-1}, \end{aligned} \tag{24}$$

where the matrices $\mathrm{P} = \mathrm{diag}\{+ + +\}$ and $\mathrm{P}' = \mathrm{diag}\{+ + -\}$. Now the (Z_2, Z_2') parities of the SM gauge bosons and the U, V gauge bosons are $(++)$ and $(+-)$, respectively. It is easy to work out the Fourier eigenmodes propagating in the bulk and see that only fields with $(++)$ parity have zero modes. In other words, only SM gauge bosons have zero modes. Both the U, V gauge bosons and all the $y-$ components are heavy Kaluza-Klein (KK) excitation. Note the second Z_2' is necessary to avoid the presence of zero modes for both SM gauge boson and the exotic $U^{\pm 2}$, $V^{\pm}$ boson at the same time.

The $SU(3)_W$ symmetry is explicitly broken to $SU(2)_L \times U(1)$ at the $y = \pi R/2$ fixed point, where the 4D quarks field are forced to live on it. The extra degree of freedom in extra-dimensional theories is the key to incorporate SM quarks into the $SU(3)_W$ symmetry. On the other hand, the lepton fields can be placed anywhere in the bulk or on either two fixed points. We choose to put the 4D lepton triplets at $y = 0$ which is a $SU(3)_W$ symmetric fixed point so that they enjoy the $SU(3)_W$ symmetry. This also avoids possible proton decay contact interactions.

One Higgs triplet **3** plus one Higgs antisextet $\bar{\mathbf{6}}$, denoted as ϕ_6, with parities

$$\begin{aligned} &\phi_3(y) = \mathrm{P}\phi_3(-y), \qquad \phi_3(y') = \mathrm{P}'\phi_3(-y'), \\ &\phi_6(y) = \mathrm{P}\phi_6(-y)\mathrm{P}^{-1}, \qquad \phi_6(y') = -\mathrm{P}'\phi_6(-y')\mathrm{P}'^{-1}. \end{aligned} \tag{25}$$

is the minimal scalar set to give viable charged fermion masses (see [7]). Another Higgs triplet $\mathbf{3}'$ with parities $(+-)$ is introduced to transmit lepton number violation essential for generating Majorana neutrino mass through one-loop diagrams [7] by a triple Higgs interaction of the type of $\mathbf{3}'^T\bar{\mathbf{6}}\mathbf{3}$. This is a 5D realization of radiative neutrino mass generation first proposed in [18]. The resulting mass matrix is necessarily of the Majorana type.

Now we have all the ingredients to write down explicitly the 5D Lagrangian density

$$\mathcal{L}_5 = -\frac{1}{2}\,\mathrm{Tr}[G_{MN}G^{MN}] + \mathrm{Tr}[(D_M\phi_6)^\dagger(D^M\phi_6)] + (D_M\phi_3)^\dagger(D^M\phi_3) + (D_M\phi_3')^\dagger(D^M\phi_3') + \delta(y) \times\left[\epsilon_{abc}\frac{f_{ij}^3}{\sqrt{M^*}}\overline{(L_i^a)^{\mathrm{c}}}L_j^b\phi_3^c + \epsilon_{abc}\frac{f_{ij}'^3}{\sqrt{M^*}}\overline{(L_i^a)^{\mathrm{c}}}L_j^b\phi_3'^c + \frac{f_{ij}^6}{\sqrt{M^*}}\overline{(L_i^a)^{\mathrm{c}}}\phi_{6\{ab\}}L_j^b + \bar{L}i\gamma^\mu D_\mu L\right] - V_0(\phi_6, \phi_3, \phi_3') - \frac{m}{\sqrt{M^*}}\phi_3^T\phi_6\phi_3' + \mathrm{H.c.} + \mathcal{L}_{\mathrm{GF}} + \text{ quark sector}, \tag{26}$$

where G_{MN}, $M, N = \{0, 1, 2, 3, y\}$ is the 5D field strength and D_M is the 5D covariant derivative. The cutoff scale M^* is introduced to make the coupling constants dimensionless. The other notations are self explanatory. The quark sector is not relevant now and will be left out. The complicated scalar potential is gauge invariant and orbifold symmetric and will not be specified since it is not needed here. To perform loop calculations, we need to specify the 5D gauge fixing term, $\mathcal{L}_{\mathrm{GF}}$, which will be exhibited later.

The fields and their parities of this model are summarized below:

$$8^\mu = \underbrace{(1,0)_{++}}_{B^\mu} + \underbrace{(3,0)_{++}}_{A^\mu} + \underbrace{(2,-3/2)_{+-} + (2,+3/2)_{+-}}_{(U,V)^\mu},$$

$$8^y = \underbrace{(1,0)_{--}}_{B^y} + \underbrace{(3,0)_{--}}_{A^y} + \underbrace{(2,-3/2)_{-+} + (2,+3/2)_{-+}}_{(U,V)^y},$$

$$\mathbf{3} = \underbrace{(2,-1/2)_{++}}_{H_{W1}} + \underbrace{(1,1)_{+-}}_{H_S},$$

$$\mathbf{3}' = \underbrace{(2,-1/2)_{+-}}_{H_{W1}'} + \underbrace{(1,1)_{++}}_{H_S'},$$

$$\bar{\mathbf{6}} = \underbrace{(3,+1)_{+-}}_{H_T} + \underbrace{(2,-1/2)_{++}}_{H_{W2}} + \underbrace{(1,-2)_{+-}}_{H_{S2}},$$

where the SM quantum numbers are $(\mathrm{SU}(2)_L, \mathrm{U}(1)_Y)$ and the subscripts label the parities P, P'. Then it is straightforward to obtain the 4D effective interaction by integrating over y and the 4D effective gauge coupling can be identified as $g_2 = \tilde{g}/\sqrt{2\pi RM^*}$. The orbifold construction is engineered such that there is no tree-level LFV in the SM gauge interactions. Thus, the success of that model remains intact. But the tree-level LFV interactions emerge in the U, V gauge interactions which are heavy KK excitation and in the Yukawa interactions.

The LFV charged current is

$$\mathcal{L}_{\mathrm{CC}} = g_2\sum_{n=1}\bar{e}_{Li}\,\gamma^\mu P_L(\mathcal{U}_{lep})_{ij}e_{Rj}^{\mathrm{c}}U_{n,\mu}^{-2} + \mathrm{H.c.} + g_2\sum_{n=1}\bar{\nu}_{Li}\,\gamma^\mu P_L(\mathcal{U}_{lep})_{ij}e_{Rj}^{\mathrm{c}}V_{n,\mu}^{-1} + \mathrm{H.c.}, \tag{27}$$

where the subscripts L and R are kept for bookkeeping. The matrices $U_{L,R}$ are used to diagonalize the charged lepton mass matrix and $\mathcal{U}_{lep} = U_L^\dagger U_R^*$ is an extra Cabibbo-Kobayashi-Maskawa (CKM)-like unitary mixing matrix for the lepton sector.

The LFV Yukawa interactions are given by

$$\mathcal{L}_{\mathrm{Y}} = \frac{1}{\sqrt{2\pi RM^*}}\sum_{n=0}\kappa_n[f_{Sij}^3\bar{e}_{L,i}^{\mathrm{c}}\nu_{L,j}H_{S,n}^+ + f_{Hij}^3(\bar{e}_{R,i}e_{L,j}H_{W1,n}^0 + \bar{e}_{R,j}\nu_{L,i}H_{W1,n}^-) - (i\Leftrightarrow j)] + \frac{1}{\sqrt{2\pi RM^*}}\sum_{n=0}\kappa_n[f_{Sij}'^3\bar{e}_{L,i}^{\mathrm{c}}\nu_{L,j}H_{S,n}'^+ + f_{Hij}'^3(\bar{e}_{R,i}e_{L,j}H_{W1,n}'^0 + \bar{e}_{R,j}\nu_{L,i}H_{W1,n}'^-) - (i\Leftrightarrow j)] + \frac{1}{\sqrt{2\pi RM^*}}\sum_{n=0}\kappa_n\{f_{Tij}^6[\bar{e}_{L,i}^{\mathrm{c}}e_{L,j}H_{T,n}^{+2} + (\bar{e}_{L,i}^{\mathrm{c}}\nu_{L,j} + \bar{\nu}_{L,i}^{\mathrm{c}}e_{L,j})H_{T,n}^+ + \bar{\nu}_{L,i}^{\mathrm{c}}\nu_{L,j}H_{T,n}^0] + f_{Hij}^6[(\bar{e}_{R,i}e_{L,j} + \bar{e}_{L,i}e_{R,j})H_{W2,n}^0 + (\bar{e}_{R,i}\nu_{L,j} + \bar{\nu}_{L,i}e_{R,j})H_{W2,n}^-] + f_{Sij}^6\bar{e}_{R,i}e_{R,j}^{\mathrm{c}}H_{S2,n}^{-2}\} + \mathrm{H.c.}, \tag{28}$$

where $\kappa_n = (\sqrt{2})^{1-\delta_{n,0}}$ and

$$f_T^6 = U_L^T f^6 U_L, \qquad f_S^6 = U_R^T f^6 U_R, \qquad f_S^{(\prime)3} = U_L^T f^{(\prime)3} U_L, \qquad f_H^{(\prime)3} = U_R^\dagger f^{(\prime)3} U_L, \qquad f_H^6 = U_R^\dagger f^6 U_L. \tag{29}$$

Note that in the new basis the symmetry of f_T and f_S are not changed.

A. $L \to l + \gamma$ transition

We begin the discussion by studying a special case that $f_6 \gg f_3$, such that $U_R \sim U_L^*$ also f_T^6, f_H^6, and f_S^6 are roughly diagonal. This hierarchical Yukawa structure is also demanded to yield the observed charged lepton mass hierarchy. In other words, all the LFV sources are in the Yukawa interaction of ϕ_3 and ϕ_3'. Since ϕ_3' has nothing to do with the charged lepton masses, we can further assume its LFV contribution is larger than ϕ_3, whose coupling is roughly $\sim(m/M_W)(f_3/f_6)$, and $f_S'^3 \sim f_H'^3$.

In general this class of decays proceeds via the one-loop diagrams. The ones involving the gauge boson $U^{\pm 2}$ and $V^{\pm}$ are suppressed by the GIM mechanism. This leaves the singly charged and neutral scalars as the only possible contributors since they both carry two units of lepton charges in the usual scheme. We thus conclude that these decays are dominated by the scalar induced $M1$ and $E1$ operators only. Therefore, they provide unique probes of the exotic scalar sector. Later we will show that in contrast $L \to 3l$ probes the gauge interactions of the model.

In this case, the leading contribution loop diagrams are shown in Fig. 1. Now, briefly discuss the gauge fixing in this model. Because the orbifold parity for ϕ_3' is chosen to be $(+-)$, it cannot develop a vacuum expectation value (VEV) and does not participate in the electroweak breaking. The Goldstone bosons consist of the y-components of gauge bosons and the proper linear combinations of ϕ_3 and ϕ_6. And the whole $\mathbf{3}'$, $H_{W1}'^0$, $H_{W1}'^{\pm}$, and H_S' are physical Higgs. So now it is straightforward to carry out loop calculation. For further details, see Appendix A and B.

The $E1$, $M1$ form factors are calculated to be:

$$f_{M1}^{Ll} = \frac{m_\mu^2}{384\pi^2 M_{S0}^2} \sum_i \left[f_{Li} + \frac{\epsilon}{24}(9f_{Ri} + 7f_{Li}) \right], \tag{30}$$

$$f_{E1}^{Ll} = \frac{m_\mu^2}{384\pi^2 M_{S0}^2} \sum_i \left[f_{Li} - \frac{\epsilon}{24}(9f_{Ri} - 7f_{Li}) \right], \tag{31}$$

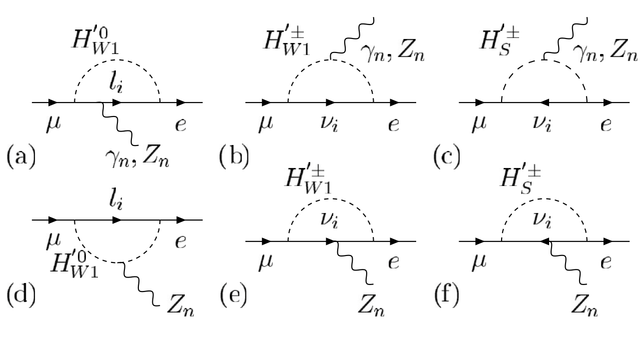

FIG. 1. The leading contributions for $\mu \to e\gamma$ and $\mu - e$ conversion in the case described in Sec. III A. The labels $n \geq 0$ indicate the KK level.

where $f_{Li} = f_{Ri}^* \equiv f_{li}^* f_{iL}$, M_{S0} is the zero mode mass of $H_S'^{\pm}$, and $\epsilon = (\pi M_{S0} R)^2 \sim \mathcal{O}(0.1)$. On arriving at the above expression, the contributions of all KK scalar excitation running in the loop have been summed. And if we drop the ϵ-terms, the resulting branch ratio can be expressed as:

$$B(L \to l + \gamma) = \frac{96\pi^3 \alpha}{G_F^2 m_\mu^4}(|f_{E1}^{Ll}|^2 + |f_{M1}^{Ll}|^2) \sim \frac{\alpha}{768\pi G_F^2 M_{S0}^4} \times \left| \sum_{i=e,\mu,\tau} f_{Li} \right|^2 \tag{32}$$

$$= 2.75 \times 10^{-6} \left(\frac{300\ \text{GeV}}{M_{S0}} \right)^4 |(f_{S,le}'^3)^* f_{S,eL}'^3 + (f_{S,l\mu}'^3)^* f_{S,\mu L}'^3 + (f_{S,l\tau}'^3)^* f_{S,\tau L}'^3|^2. \tag{33}$$

Because the Yukawa couplings of triplet scalars are antisymmetric, the $L \to l + \gamma$ processes have the following forms:

$$B(\mu \to e + \gamma) = 2.75 \times 10^{-6} \left(\frac{300\ \text{GeV}}{M_{S0}} \right)^4 |(f_{S,e\tau}'^3)^* f_{S,\mu\tau}'^3|^2, \tag{34}$$

$$B(\tau \to e + \gamma) = 2.75 \times 10^{-6} \left(\frac{300\ \text{GeV}}{M_{S0}} \right)^4 |(f_{S,e\mu}'^3)^* f_{S,\mu\tau}'^3|^2, \tag{35}$$

$$B(\tau \to \mu + \gamma) = 2.75 \times 10^{-6} \left(\frac{300\ \text{GeV}}{M_{S0}} \right)^4 |(f_{S,e\mu}'^3)^* f_{S,e\tau}'^3|^2. \tag{36}$$

We have taken $M_3' = 300$ GeV as the reference point. If all of the Yukawa couplings are real and none of them vanishes, their ratios can be further simplified to:

$$B(\mu \to e + \gamma):B(\tau \to e + \gamma):B(\tau \to \mu + \gamma) = \frac{1}{|f_{S,e\mu}'^3|^2} : \frac{1}{|f_{S,e\tau}'^3|^2} : \frac{1}{|f_{S,\mu\tau}'^3|^2}. \tag{37}$$

At this point one can use the data $B(\mu \to e + \gamma) < 1.2 \times 10^{-11}$ [19] to obtain the constrain $|(f_{S,e\tau}'^3)^* f_{S,\mu\tau}'^3| < 2.1 \times 10^{-3}$. This is consistent with the expectation from the study of neutrino mass in this as given in [7]. There it was found that the Yukawa coupling $f_{e\mu}'^3$ has to be $\lesssim 10^{-2}$ and the f's exhibit the pattern $f_{e\mu}'^3 > f_{e\tau}'^3 > f_{\mu\tau}'^3$. Hence it is reasonable for $\mu \to e + \gamma$ to occur at a rate less than 2 orders of magnitude below current level. Indeed in the $SU(3)_W$ model we can link the various $L \to l\gamma$ transition branch ratios to the light neutrino mass matrix elements. Assuming that the light neutrino mass is mostly coming from the one-loop quantum correction involving the zero modes of ϕ_3' and ϕ_6, we have the prediction :

$$B(\mu \to e + \gamma):B(\tau \to e + \gamma):B(\tau \to \mu + \gamma) \sim \left(\frac{m_\mu}{m_\tau}\right)^4 m_{13}m_{23}:m_{12}m_{23}:m_{12}m_{13}, \quad (38)$$

where m_{ij} is the (ij) entry of the light neutrino mass matrix. Interestingly the model naturally accommodates an active neutrino mass matrix of the inverted hierarchy type as follows:

$$m \sim \begin{pmatrix} \epsilon & 1 & 1 \\ 1 & \epsilon & \epsilon \\ 1 & \epsilon & \epsilon^2 \end{pmatrix}, \quad (39)$$

where $\epsilon \sim 0.1$. From the above equations, we see that $\mu \to e\gamma$ is suppressed compared to the $\tau \to l\gamma$ decays. This is a striking feature of the model.

B. $\mu - e$ conversion

The $\mu - e$ conversion in nuclei will be dominated by the virtual photon exchange. Compared to $\mu \to e\gamma$ it has additional contributions from the anapole terms. The corresponding photon $E0$, $M0$ form factors can be derived as:

$$\tilde{f}_{E0}(-k^2) = \frac{m_\mu^2}{576\pi^2 M_{S0}^2} \sum_{i=e,\mu,\tau} \left[f_{Li} + \frac{\epsilon}{24}(3f_{Ri} + f_{Li}) - \frac{6\epsilon}{\pi^2}(f_{Ri} + f_{Li}) \sum_{n=1}^{\infty} \frac{G(\delta_n, x_i)}{(2n-1)^2} \right], \quad (40)$$

$$\tilde{f}_{M0}(-k^2) = \frac{m_\mu^2}{576\pi^2 M_{S0}^2} \sum_{i=e,\mu,\tau} \left[f_{Li} + \frac{\epsilon}{24}(3f_{Ri} - f_{Li}) - \frac{6\epsilon}{\pi^2}(f_{Ri} - f_{Li}) \sum_{n=1}^{\infty} \frac{G(\delta_n, x_i)}{(2n-1)^2} \right], \quad (41)$$

where $\delta_n = (-k^2)/M_{Hn}^2$, $x_i = m_i^2/(-k^2)$, $i = e, \mu, \tau$, and

$$G(\delta, x) = -\ln\delta - \ln x + \frac{1}{3} - 4x + (1-2x) \times \sqrt{1+4x}\ln\frac{\sqrt{4x+1}-1}{\sqrt{4x+1}+1}. \quad (42)$$

As expected, the principal contribution is from the Fig. 1(c) with the $H_S'^{\pm}$ zero mode running in the loop. The logarithmic enhancements in $G(\delta, x)$, is due to the exchange of neutral scalars, $H_{W1}'^0$ [see Fig. 1(a)]. Although they are suppressed by the KK masses we find them to be compatible to the charged singlet contribution.

In this process $-k^2 \sim m_\mu^2$ and G has the following limits

$$G_{e,n} \sim -\ln\frac{m_\mu^2}{M_{Hn}^2} + \frac{1}{3}, \quad G_{\mu,n} \sim -\ln\frac{m_\mu^2}{M_{Hn}^2} - 1.515, \quad G_{\tau,n} \sim -\ln\frac{m_\mu^2}{M_{Hn}^2} - 6.978. \quad (43)$$

The KK sum of these logarithmic enhancements are finite:

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \ln\frac{m_\mu^2}{M_{Hn}^2} = \frac{\pi^2}{8}\ln(m_\mu R)^2 - 0.8362. \quad (44)$$

So the desired anapole form factors can be expressed as

$$\tilde{f}_{E0}(m_\mu^2) = \frac{m_\mu^2}{576\pi^2 M_{S0}^2} \sum_{i=e,\mu,\tau} \left\{ f_{Li} + \frac{\epsilon}{24}(3f_{Ri} + f_{Li}) + \frac{3\epsilon}{4}(f_{Ri} + f_{Li})[\ln(m_\mu R)^2 + \eta_i] \right\}, \quad (45)$$

$$\tilde{f}_{M0}(m_\mu^2) = \frac{m_\mu^2}{576\pi^2 M_{S0}^2} \sum_{i=e,\mu,\tau} \left\{ f_{Li} - \frac{\epsilon}{24}(3f_{Ri} - f_{Li}) + \frac{3\epsilon}{4}(f_{Ri} - f_{Li})[\ln(m_\mu R)^2 + \eta_i] \right\}, \quad (46)$$

$\{\eta_e, \eta_\mu, \eta_\tau\} = \{-1.011, 0.837, 6.300\}$. Again, since $f_S'^3$ is antisymmetric, only $f_{L\tau} = f_{R\tau}^*$ can contribute. For simplicity we assume there is no new CP violation in the scalar sector; then $f_L = f_R$ and the $\mu - e$ conversion rate in ${}^{48}_{22}Ti$ can be expressed as:

$$B_{\rm conv}^{\gamma} \sim 0.01B(\mu \to e + \gamma) \quad (47)$$

if taking $1/R = 2$ TeV and $M_S = 300$ GeV as a reference point. It is also possible to have extra contributions from KK photon and KK Z excitation, Figs. 1(a)–1(f). One will need to take care of the KK number conservation in the scalar-scalar-gauge boson vertices and sum over all the possible combinations. But generally speaking, their contributions are further suppressed by $(m_\mu R)^2 < 2 \times 10^{-9}$ compared to the photon zero mode and can be safely ignored.

The relation of Eq. (47) is based on the assumption that $f_6 \gg f_3$ and ϕ_3' is the dominate LFV source. However, we should point out that if f_3 is not so small the neutral scalar zero modes can make $\mu \to e\gamma$ and $B_{\rm conv}^{\gamma}$ compatible and deviates a lot from the pure photonic dipole prediction, Eq. (23).

Again, this demonstrates that $L \to l\gamma$ and $\mu - e$ conversions are very important for us to understand the Yukawa structure in the $SU(3)_W$ model.

The question now arises about the photonic dipole and anapole contribution to $\mu \to 3e$. The answer lies in Eqs. (30), (31), (40), (41). We estimated that

$$B(\mu \to 3e) < 0.04B(\mu \to e\gamma). \quad (48)$$

This prediction is not very sensitive to what the Yukawa pattern is. Moreover, the decays $L \to 3l$ have overwhelming contributions from other sources of new physics in the model to which we shall turn our attention to next.

C. $L \to 3l$

A characteristic of the model is the existence of double charged gauge bosons with LFV couplings. This will induce $\mu \to 3e$ like processes for the τ. In addition there are also KK scalars H_T and H_0 which has LFV Yukawa couplings which are largely unknown. The Feynman diagrams for the $L \to 3l$ decays are depicted in Fig. 2. Since the Yukawa couplings are totally unknown, we will postpone the discussion of the contributions from scalars and look at the branch ratios, normalized to $B(\tau \to e\nu_\tau\bar{\nu}_e)$, mediated by $U^{\pm 2}$ gauge boson alone first:

$$B(\tau \to 3\mu) = \mathcal{F} \times (|\mathcal{U}_{\tau\mu}|^2 + |\mathcal{U}_{\mu\tau}|^2)|\mathcal{U}_{\mu\mu}|^2, \quad (49)$$

$$B(\tau \to 3e) = \mathcal{F} \times (|\mathcal{U}_{\tau e}|^2 + |\mathcal{U}_{e\tau}|^2)|\mathcal{U}_{ee}|^2, \quad (50)$$

$$B(\tau \to \bar{\mu}ee) = \mathcal{F} \times (|\mathcal{U}_{\tau\mu}|^2 + |\mathcal{U}_{\mu\tau}|^2)|\mathcal{U}_{ee}|^2, \quad (51)$$

$$B(\tau \to \mu\mu\bar{e}) = \mathcal{F} \times (|\mathcal{U}_{\tau e}|^2 + |\mathcal{U}_{e\tau}|^2)|\mathcal{U}_{\mu\mu}|^2, \quad (52)$$

$$B(\tau \to \mu e\bar{e}) = \frac{\mathcal{F}}{8}(|\mathcal{U}_{\tau e}|^2 + |\mathcal{U}_{e\tau}|^2)(|\mathcal{U}_{e\mu}|^2 + |\mathcal{U}_{\mu e}|^2), \quad (53)$$

$$B(\tau \to e\mu\bar{\mu}) = \frac{\mathcal{F}}{8}(|\mathcal{U}_{\tau\mu}|^2 + |\mathcal{U}_{\mu\tau}|^2) \times (|\mathcal{U}_{e\mu}|^2 + |\mathcal{U}_{\mu e}|^2). \quad (54)$$

where $\mathcal{F} = (M_W\pi R)^4/16 = 1.56 \times 10^{-5}(2\ \mathrm{TeV}/1/R)^4$. From the analysis given in Sec. II, we know all scalar operators give positive contribution. So even though we know nothing about the Yukawa couplings, we can still derive an interesting lower bond from the unitarity of $\mathcal{U}_{lep}$

$$B(\tau \to 3e) \geq \mathcal{F} \times |\mathcal{U}_{ee}|^2(1 - |\mathcal{U}_{ee}|^2) \quad (55)$$

for a given $1/R$.

If one wants to keep compactification scale $1/R$ low, say $\sim$1.5 TeV, then we would require $|\mathcal{U}_{ee}|$ to be either close to zero or one. Furthermore, if we take the upper bound of $1/R < 5$ TeV derived from unification seriously we obtain

$$B(\tau \to 3e) > 8.0 \times 10^{-7}|\mathcal{U}_{ee}|^2(1 - |\mathcal{U}_{ee}|^2). \quad (56)$$

On the other hand, if we assume that the bilepton gauge boson exchange is the dominating flavor changing neutral current (FCNC) source, another interesting upper bond can be derived:

$$B(\tau \to 3e) < \frac{\mathcal{F}}{4} = 3.9 \times 10^{-6}\left(\frac{2\ \mathrm{TeV}}{1/R}\right)^4 \quad (57)$$

with $|\mathcal{U}_{ee}| = 1/\sqrt{2}$ in Eq. (55). Actually, if all the LFV Yukawa couplings are associated with ϕ_3' as discussed in the previous two subsections, the tree-level bilepton scalar contributions to $\tau \to 3e$ vanish due to the antisymmetry of the Yukawa couplings. However, the present experimental limit, $1 - 3 \times 10^{-7}$ [20] will indicate that the compactification radius is closer to the upper limit of 5 TeV^{-1} for this particular case.

IV. 5D SU(5) MODEL

The orbifold $SU(3)_W$ model discussed above has many interesting and novel features; however, the fact that quarks and leptons have to be treated differently is an obstacle towards complete unification. It is a natural attempt to further unify the quarks and leptons in a larger GUT group. The simplest group for that is SU(5). Now all fermions are on equal footing and can be clustered into two SU(5) representations, i.e., $\Psi_{\bar{5}} = \{d^c, L\}$, $\Psi_{10} = \{Q, u^c, e^c\}$.

Similar to the $SU(3)_W$ model, the model is embedded in the background geometry of $S_1/Z_2 \times Z_2'$ orbifold. The bulk SU(5) gauge symmetry is broken to the SM by orbifold parities, with parity matrices diag$\{+ + + + +\}$ and diag$\{- - - + +\}$ for Z_2 and Z_2' transformations, respectively. These are generalizations of the $SU(3)_W$ case.

Since no right-handed neutrinos are added, neutrino masses can be generated through quantum correction by using either **10** or **15** bulk scalars plus the $\bar{\mathbf{5}}'(\mathbf{10}/\mathbf{15})\bar{\mathbf{5}}$ interaction mandated by breaking to the SM gauge group. The orbifold parities of **10** or **15** bulk scalars are determined to be $(++)$ by considerations of proton decay. They split into the following components:

$$\mathbf{15}_s(++) = P_{15}\left(6, 1, -\frac{2}{3}\right)_{++} + T_{15}(1, 3, 1)_{++} + C_{15}\left(3, 2, \frac{1}{6}\right)_{+-},$$

$$\mathbf{10}_a(++) = P_{10}\left(\bar{3}, 1, -\frac{2}{3}\right)_{++} + S_{10}(1, 1, 1)_{++} + C_{10}\left(3, 2, \frac{1}{6}\right)_{+-}.$$

A careful analysis shows that by using **15**(**10**) the resultant neutrino mass matrix favor the normal (inverted) hierarchy [8]. It was also found that extra fine tuning efforts were needed to obtain phenomenologically acceptable neutrino mass patterns by using **10** alone; so we will only discuss the case which implements **15**.

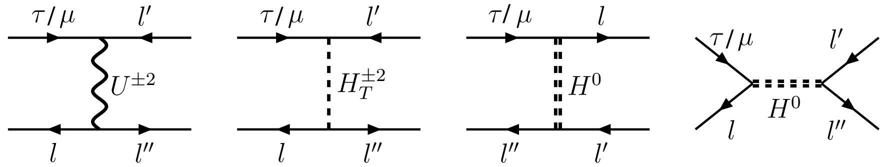

FIG. 2. Feynman diagrams which lead to $\tau(\mu) \to 3l$ processes.

The P components induce tree-level $K^0 - \bar{K}^0$ mixing. To satisfy the experimental constraints, it is required that $M_P > 10^5$ GeV. On the other hand, the two bulk Higgs in **5**, **5**$'$ which are responsible for the SM electroweak symmetry breaking share the same $(++)$ parities as **15**. The brane Yukawa interaction term is easily constructed to be

$$\mathcal{L}_Y = \delta(y)\left[\frac{\tilde{f}_{ij}^{15}}{\sqrt{M^*/2}}\overline{\psi}_{\bar{5}i}^{\{A\}c}\psi_{\bar{5}j}^{\{B\}}\phi_{15}^{\{AB\}} + \text{H.c.}\right], \tag{58}$$

where A, B are the SU(5) symmetry indices. It can be seen that Eq. (58) contains the necessary LFV source to generate neutrino Majorana masses. The neutrino mass matrix elements are proportional to $(\mathcal{M})_{ij}^{\nu} \propto \sum_k m_k f_{ik}'^{5} f_{jk}^{15}$ where i, j, k are the generation indices and m_k is the mass of k-charged lepton running in the loop.

The extra Higgs doublet in the **5**$'$ is good for gauge unification. By adding additional decuplet bulk fermion pair with $(+-)$ parity and mass around $10 - 20$ TeV, the unification is achieved at $3 \times 10^{16} - 10^{15}$ GeV or equivalently $1/R \sim 10^{14}$ GeV. The high scale unification or tiny radius of extra dimension makes KK excitation decouple from most phenomenological studies and basically we only need to consider the zero modes.

Below unification scale or equivalently the low energy 4D effective theory is a two Higgs doublets like model. In general the two Yukawa patterns are not aligned which can lead to severe tree-level charged neutral flavor changing (FCNC) interaction. A Z_2 symmetry is usually assumed to forbid such tree-level FCNC [21]. In this model, there is no such freedom since the Yukawa patterns are determined by the geometrical setup. The Ψ_{10} of the first two generations are assigned to be bulk fields and the other fermion fields, Ψ_{10}^3 and $\Psi_{\bar{5}}^{1,2,3}$, are localized at the $y = 0$ brane. In doing so, the salient SU(5) prediction of m_b/m_τ ratio is preserved and give small hierarchy patterns in the Yukawa couplings of both **5**, **5**$'$ scalars, i.e.,

$$y_d \propto \begin{pmatrix} \delta & \delta & 1 \\ \delta & \delta & 1 \\ \delta & \delta & 1 \end{pmatrix}, \qquad y_u \propto \begin{pmatrix} \delta^2 & \delta^2 & \delta \\ \delta^2 & \delta^2 & \delta \\ \delta & \delta & 1 \end{pmatrix}. \tag{59}$$

Because of volume dilution factor we get $\delta \sim 0.1$ which measures the amount of overlap between brane and bulk fields. The specific Yukawa pattern above successfully generates mass and mixing hierarchy of charged fermions:

$$m_b:m_s:m_d = m_\tau:m_\mu:m_e \sim 1:\delta:\delta^2, \tag{60}$$

$$m_t:m_c:m_u \sim 1:\delta^2:\delta^4, \quad (V_{us}, V_{cb}, V_{ub}) \sim (\delta, \delta, \delta^2). \tag{61}$$

The rotation from weak to mass eigenbasis simultaneously diagonalizes the two Higgs doublets Yukawa couplings. Thus, we do not have the FCNC problem due to mixing between two Higgs doublets.

Instead, now tree-level LFV processes can be mediated by the triplet component T_{15} in **15**. The only important ones are the $\mu \rightarrow 3e$ like processes, see Fig. 3.

An explicit calculation give the branching ratio of $\mu \rightarrow 3e$:

$$Br(\mu \rightarrow 3e) = \frac{2|f_{11}^{15\dagger} f_{12}^{15}|^2}{g_2^4(\pi R M^*)^2}\left(\frac{M_W}{M_T}\right)^4. \tag{62}$$

The mass difference $\triangle M_K^P$ in $K^0 - \overline{K}^0$ mixing arises from $P_{10,15}$ and can be used to eliminate the ambiguity of absolute strength of Yukawa couplings. The ratio of Yukawa couplings can be replaced by the ratio of the corresponding elements in $\mathcal{M}_\nu$. Since only the **15** Higgs is used, we have

$$Br(\mu \rightarrow 3e) \sim 3.02 \times 10^{-16}\left(\frac{\triangle m_K^P}{\triangle m_K}\right)^2\left(\frac{M_P}{M_T}\right)^4 \times \left[\frac{2m_{11}m_{12}}{m_{11}m_{22} + (2\frac{m_e}{m_\mu}m_{12})^2}\right]^2. \tag{63}$$

It is straightforward to extend the analysis to $\tau \rightarrow 3l$ transitions. Assuming that the hierarchy of the elements of neutrino mass matrix is smaller than factor 100, this model predicts

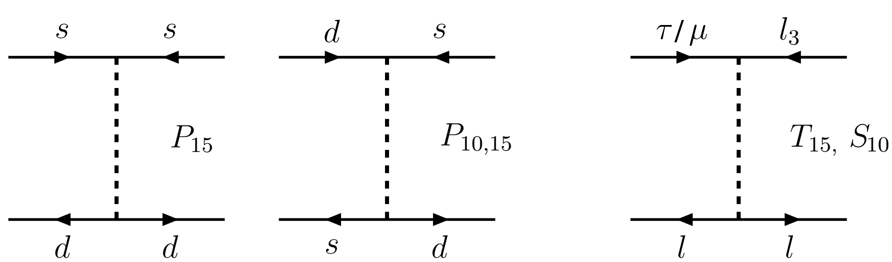

FIG. 3. Feynman diagrams for tree-level $K^0 - \bar{K}^0$ mixing and $\mu \rightarrow 3e$ process.

$$Br(\mu \to 3e):Br(\tau \to 3e):Br(\tau \to 3\mu):Br(\tau \to \mu ee):Br(\tau \to e\mu\mu)$$
$$\sim \frac{m_{12}^2}{m_{22}^2}:\left(\frac{m_\mu}{m_\tau}\right)^4\frac{m_{13}^2}{m_{22}^2}:\left(\frac{m_e}{m_\tau}\right)^4\frac{m_{23}^2}{m_{11}^2}:\left(\frac{m_\mu}{m_\tau}\right)^4\frac{m_{23}^2}{m_{22}^2}:\left(\frac{m_e}{m_\mu}\right)^4\frac{m_{12}^2}{m_{11}m_{22}}. \tag{64}$$

The photonic form factors due to $T^{\pm 2}$, $T^{\pm}$ scalar one-loop diagrams can be obtained:

$$f_{M1}^{\mu e} = f_{E1}^{\mu e} = -\frac{m_\mu^2}{16\pi^2 M_T^2}\frac{5f_{i\mu}^{15}(f_{ie}^{15})^*}{24}, \tag{65}$$

$$\tilde{f}_{M0}^{\mu e} = \tilde{f}_{E0}^{\mu e} = \frac{m_\mu^2}{16\pi^2 M_T^2}\frac{f_{i\mu}^{15}(f_{ie}^{15})^*}{6}\left[G(\delta, x_i) - \frac{1}{2}\right]. \tag{66}$$

The chiral structure in the above result is easily understood because only the lepton doublets interact with triplet scalar.

Because of the factor $(m_\mu/M_T)^4$, the $\mu \to e\gamma$ process is strongly suppressed. Taking $M_T = 10^5$ GeV, the $\mu Ti \to eTi$ conversion rate is estimated to be $\sim 1.2 \times 10^{-14}|f_{i\mu}^{15}(f_{ie}^{15})^*|^2$ where we have kept only logarithmic terms which is sufficient for an order of magnitude estimate. The experimental bound is 3.6×10^{-11} [19] which is not stringent. On the other hand the recently proposed experiment aimed at detecting a signal at the $10^{-16} - 10^{-17}$ will be very encouraging. Even a negative result will provide stringent constrain on the otherwise unknown Yukawa couplings.

V. SPLIT FERMION MODEL

An interesting scenario was introduced by [22] to solve the charged fermion masses hierarchy problem. The basic idea is to postulate that fermions are bulk fields and they interact with a nondynamic background scalar potential. In the 5D version the bulk fermions are vectorlike, but only one of the chiral zero modes will be localized at the zero of the background potential modulated by the 5D mass terms. The chirality of the zero mode is determined by the sign of slope of the background potential at the zero point. The fermion zero modes are given a Gaussian profile in the fifth dimension and each has its own unique position in the extra dimension. The widths are controlled by the potential slope at the localized position. For simplicity, we will assume a universal width for all the SM fermions. The 5D fermion excitations are vectorlike and will be located at the same position of their zero modes. Roughly speaking, the energy gap is $\sim 1/(\text{Width}) \gg 1/R \gg M_W$ and they decouple from the phenomenology we are interested in. Since the SM fermions are scattered over the fifth dimension, to preserve the gauge interaction universality the SM gauge fields are forced to be bulk fields too. To illustrate the basic physics involved it suffices to build a model on an S_1/Z_2 orbifold so that one can remove the unwanted y-components of gauge bosons zero modes which are identified with the SM gauge bosons. However, to break electroweak symmetry, a dynamic bulk Higgs is necessary.

To be more concrete we take the extra dimension to be the interval $y \in [-\pi R, \pi R]$ and the fermions are fixed in different positions z_i in this interval. The usual Kaluza-Klein ansatz is invoked that any bulk field factorizes into a 4D field times a 5D wave function. Thus, for the fermion field we have $\psi_i(x, y) = g(z_i, y)\psi(x)$ where $g(z_i, y)$ is taken to be Gaussian distribution in the fifth dimension:

$$g(z_i, y) = \frac{1}{(\pi\sigma_G^2)^{1/4}}\exp\left[-\frac{(y - z_i)^2}{2\sigma_G^2}\right],$$

where σ_G is the universal width of Gaussian distribution. If $\sigma_G \ll R$, g acts like the Dirac delta function. Effective 4D interactions are obtained by integrating out the y direction. Then any pair fermions get an exponential suppression

$$g(z_1, y)g(z_2, y) = \exp\left[-\frac{(z_1 - z_2)^2}{4\sigma_G^2}\right]g\left(\frac{z_1 + z_2}{2}, y\right)$$

as a result of integrating over Gaussian functions. Therefore, the linear displacement between left-handed and right-handed fermions in the fifth dimension translates into exponential Yukawa hierarchy in 4D theory. One set of solutions for the quarks positions have been found numerically that can accommodate the mass hierarchy and the CKM mixing [23]. To accommodate the CP violation, the overall Yukawa coupling strength of up and down type quarks must be different [24,25]. Although a fundamental theory of where to place the fermions are located is lacking, we have at least one realistic solution for where the quarks are located in the extra dimension. For the lepton sector, we do not have enough constraints to pin down the solution. As pointed out by [25,26], the FCNC interaction is induced geometrically where phenomenological con-

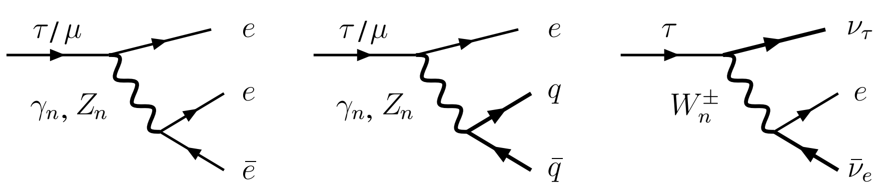

FIG. 4. Flavor changing due to KK gauge bosons.

straint in quark sector have also been discussed. In fact, flavor violation is generic in any multiposition models. This comes from the fact that weak to mass eigenbasis rotations can only make the SM interactions (zero modes) flavor diagonal. The KK modes cannot be simultaneously diagonalized.

In general, the LFV can be discussed independently of the neutrino sector. The 4D effective LFV Lagrangian can be expressed as

$$\begin{aligned}\mathcal{L}^{LFV} = &\sum_n \left(\frac{\kappa_n g_2}{\cos\theta}\right)\bar{l}_i\gamma^\mu[g_L U^L_{n(ij)}\hat{L} + g_R U^R_{n(ij)}\hat{R}]l_j Z^n_\mu \\ &- \sum_n (\kappa_n e)\bar{l}_i\gamma^\mu[U^L_{n(ij)}\hat{L} + U^R_{n(ij)}\hat{R}]l_j A^n_\mu \\ &+ \sum_n \left(\frac{\kappa_n g_2}{\sqrt{2}}\right)\bar{l}_i\gamma^\mu U^L_{n(ij)}\hat{L}\nu_j W^{n-}_\mu + \text{H.c.},\end{aligned} \tag{67}$$

where A^n, Z^n, and W^n are the nth KK excitation of the photon, Z, and W bosons and $g_{L/R} = T_3(l) - Q_l\sin^2\theta_W$. The matrices $U_n^{L/R}$ are a combination of the unitary transformations $V_{L/R}$ that take the lepton weak eigenstates to their mass eigenstates and the cosine weighting of the nth KK modes. Explicitly, they are

$$U_n^{L/R} = V^\dagger_{L/R}\mathrm{diag}\left(\cos\frac{nz_1^{L/R}}{R}, \cos\frac{nz_2^{L/R}}{R}, \cos\frac{nz_3^{L/R}}{R}\right)V_{L/R} \tag{68}$$

for $n = 0$ all the cosine factors become one and the interactions reduce to SM as expected. It is clear that the $U^{L/R}$ is no more diagonal nor unitary in general.

In this model, the $\tau \to 3l$ decay and $\mu - e$ conversion happen at tree level, see Fig. 4. The corresponding effective LFV couplings are:

$$g_3 = (2M_W R)^2(\sin^2\theta_W + g_R^2/\cos^2\theta_W)\sum_{n=1}\frac{U^{R*}_{n,ei}U^R_{n,i\mu}}{n^2}, \tag{69}$$

$$g_4 = (2M_W R)^2(\sin^2\theta_W + g_L^2/\cos^2\theta_W)\sum_{n=1}\frac{U^{L*}_{n,ei}U^L_{n,i\mu}}{n^2}, \tag{70}$$

$$g_5 = (2M_W R)^2(\sin^2\theta_W + g_R g_L/\cos^2\theta_W)\sum_{n=1}\frac{U^{R*}_{n,ei}U^L_{n,i\mu}}{n^2}, \tag{71}$$

$$g_6 = (2M_W R)^2(\sin^2\theta_W + g_L g_R/\cos^2\theta_W)\sum_{n=1}\frac{U^{L*}_{n,ei}U^R_{n,i\mu}}{n^2}, \tag{72}$$

and the v, a, v_q, and a_q can be obtained in a similar way. Also the lepton universality is broken due to flavor dependent couplings in the KK gauge interaction. We refer the reader to [26] for a detailed analysis.

The leading $\mu \to e\gamma$ contribution comes from the one-loop corrections. We need to fix the gauge before proceeding. The necessary details of 5D gauge fixing are collected in Appendix A. After properly identifying the Goldstone boson, the usual 4D R_ξ gauge technique can be straightforwardly applied here. Note that the Yukawa couplings of the physical KK scalars are suppressed by the factor of $\sim(m_l R)/n$. Although there is residual GIM cancellation in the KK gauge boson interaction, we expect the leading LFV are from KK gauge interaction.

The LFV photonic form factors due to KK gauge boson and their Goldstone boson can be calculated. The KK W bosons' contribution to the photonic form factors are given by:

$$f^W_{M1} = f^W_{E1} = \frac{7}{24}\frac{g_2^2}{16\pi^2}\sum_{n=1}\frac{(m_\mu R)^2}{n^2}U^{L*}_{n,ie}U^L_{n,i\mu}, \tag{73}$$

$$\tilde{f}^W_{M0} = \tilde{f}^W_{E0} = \frac{23}{72}\frac{g_2^2}{16\pi^2}\sum_{n=1}\frac{(m_\mu R)^2}{n^2}U^{L*}_{n,ie}U^L_{n,i\mu}, \tag{74}$$

and for the KK Z bosons they are

$$\begin{aligned}f^Z_{M1/E1} = &\frac{(m_\mu R)^2}{8\pi^2}\frac{g_2^2}{\cos^2\theta}\sum_{n=1}\sum_i\frac{1}{n^2}\Big\{-\frac{1}{3}[g_L^2 U^{L*}_{n,ie}U^L_{n,i\mu} \\ &\pm g_R^2 U^{R*}_{n,ie}U^R_{n,i\mu}] + \frac{m_i}{m_\mu}g_L g_R[U^{L*}_{n,ie}U^R_{n,i\mu} \\ &\pm U^{R*}_{n,ie}U^L_{n,i\mu}]\Big\},\end{aligned} \tag{75}$$

$$\begin{aligned}\tilde{f}^W{}_{E0/M0} = &-\frac{(m_\mu R)^2}{24\pi^2}\frac{g_2^2}{\cos^2\theta}\sum_{n=1}\sum_i\frac{1}{n^2}\Big\{\Big[G(\delta_n, x_i) + \frac{1}{2}\Big] \\ &\times[g_L^2 U^{L*}_{n,ie}U^L_{n,i\mu} \pm g_R^2 U^{R*}_{n,ie}U^R_{n,i\mu}]\Big\}.\end{aligned} \tag{76}$$

Similar contributions from KK photons can be easily read from the above by replacing $(g_2/\cos\theta) \to e$, $g_L \to 1$, and $g_R \to 1$.

These photonic form factors give extra contribution to $\mu \to 3e$ and $\mu - e$ conversion processes but cannot compete with those tree-level KK gauge boson exchanging diagrams. However they are the sole sources of new physics for the $L \to l + \gamma$ process.

In addition to the usual ignorance with regard to Yukawa coupling there are more unknowns in the lepton locations and the Gaussian widths. *Ad hoc* simplifying assumptions have to be made. Hence, this kind of model suffers from a lack of predictive power in LFV studies. More data such as the scale of neutrino mass and more complete knowledge of the neutrino mixing matrix will help greatly. However, we can extract some generic features for these kind of models as follow:

(1) $\mu/\tau \to 3l$ and $\mu Ti \to eTi$ (or $\tau \to l +$ hadrons) will happen at tree level from the exchange of KK scalars, photons, and Z bosons.
(2) $L \to l\gamma$ proceeds at the one-loop level and hence is expected to be suppressed compared to the previous modes.
(3) Violation of lepton universality will occur. The best signal will be to look for the violation in $W \to l_i \nu_i$ decays [26]. Unfortunately a more quantitative statement about the level of the effect eludes us for now.

VI. CONCLUSION

We have studied and reviewed LFV processes in 5D gauge models that are related to neutrino mass generation or address the flavor problem. Specifically we focus on two complete models which generate neutrino masses radiatively. This allows us to see in detail how the two issues can be related. The models are based on $SU(3)_W$ and SU(5) 5D unification. They give rise to different neutrino mass patterns [7,8]; thus, it is not surprising that they give different prediction for LFV. The $SU(3)_W$ model has a unification scale at ~TeV and makes essential use of bileptonic scalars. It also contains characteristic doubly charged gauge bosons. The SU(5) model is a 5D orbifold version of the usual GUT. The unification scale is much higher at 10^{15} GeV. The important ingredient for LFV and neutrino masses is the **15** Higgs representation. The triplet Higgs of this model plays the crucial role here.

We found that for the $SU(3)_W$ model the rare τ decays are much more enhanced compare to their counterpart μ decays. Among the $\tau \to l + \gamma$ decays the largest mode is the $\mu + \gamma$. Even for this mode we expect it to be $<10^{-14}$ which is much lower than current experimental reach.

The decay modes $\tau \to 3l$ have a better chance of being observed. This stems from the fact that they are tree-level processes induced by the bileptonic gauge bosons or scalars. Since they are KK modes they have high masses controlled by the extra dimension compactification radius which is ≤ 5 TeV from consistency and unification considerations. An order of magnitude improvement on the current limit will be valuable information on the unknown Yukawa couplings.

For the orbifold 5D SU(5) model the muon to electron conversion in nuclei can be within the experimental capability of the proposed experiment at Brookhaven National Laboratory [27]. As in the previous model $\mu \to e + \gamma$ will not be observable. This is very different from the conventional 4D unificational models.

The split fermion model also have the characteristic of $L \to 3l$ and $\mu \to e$ conversion dominating over $L \to l\gamma$. We cannot be more quantitative due to proliferation of unknown parameters. This model has lepton universality violation which is not present in the previous two models. This can serve as a differentiating tool.

It is clear that in order to unravel the physics behind the flavor problem all modes of LFV must be searched for. The usual 4D supersymmetric model will favor $L \to l\gamma$ where as the 5D models prefer $L \to 3l$ and/or $\mu \to e$ conversion. To this we add lepton universality test as a probe.

ACKNOWLEDGMENTS

W. F. C. thanks Y. Okada for helpful discussion and is grateful to the Institute of Theoretical Physics, Chinese Academy of Sciences, for their kind hospitality where part of the work has been completed. This work is also supported in part by the Natural Science and Engineering Council of Canada.

Note added in proof.—After completion of the paper it came to our notice that a separate discussion of LFV in extra dimension models was also given by [30].

APPENDIX A: $SU(2) \times U(1)$ IN 5D S_1/Z_2 MODEL WITH A BRANE AT $y = 0$

Now we present the gauge fixing scheme used for the 5D electroweak interaction with one bulk Higgs doublet. For simplicity the background geometry is S_1/Z_2. The fifth gamma matrix was chosen to be $\gamma^y = i\gamma^5$. The 5D Lagrangian is

$$\mathcal{L}_5 = -\frac{1}{4}F^{MN}F_{MN} - \frac{1}{4}G^{(a),MN}G^{(a)}_{MN} + (D_M\Phi)^\dagger(D^M\Phi) + \cdots, \tag{A1}$$

where

$$F_{MN} = \partial_M B_N - \partial_N B_M,$$
$$G^{(a)}_{MN} = \partial_M A^{(a)}_N - \partial_N A^{(a)}_M + \frac{\tilde{g}_2}{\sqrt{M^*}}\epsilon^{abc}A^{(b)}_M A^{(c)}_N,$$
$$D_M = \partial_M - i\frac{\tilde{g}_2}{\sqrt{M^*}}\frac{\tau^{(a)}}{2}A^{(a)}_M - i\frac{\tilde{g}_1 Y}{\sqrt{M^*}}B_M.$$

B and A stand for the U(1) hyper charge and SU(2) gauge fields, respectively. In this convention, $Q = T_3 + Y$. We adopt the usual conventions: $W^M_\pm = \frac{1}{\sqrt{2}}(A^M_1 \mp iA^M_2)$, $P^M(\text{hoton}) = (c_W B^M + s_W A^M_3)$, and $Z^M = (c_W A^M_3 - s_W B^M)$, or $B^M = (c_W P^M - s_W Z^M, A^M_3 = c_W Z^M + s_W P^M)$, where $c_W = \tilde{g}_2/\sqrt{\tilde{g}_1^2 + \tilde{g}_2^2}$ and $s_W = \tilde{g}_1/\sqrt{\tilde{g}_1^2 + \tilde{g}_2^2}$. $\tilde{g}_5 = \sqrt{\tilde{g}_1^2 + \tilde{g}_2^2}$ are introduced to simplify the notation. The symmetry breaking pattern is same as in the usual 4D SM. The bulk Higgs doublet acquires a nonzero VEV after spontaneous symmetry breaking (SSB),

$$\Phi = \begin{pmatrix} h^+ \\ \frac{\tilde{v}_b + h^0}{\sqrt{2}} \end{pmatrix}, \qquad h^0 = \phi^0 + i\chi^0. \tag{A2}$$

The generalized linear R_ξ gauge fixing is introduced [28] (other schemes can be found in [29]),

$$\mathcal{L}_{GF} = -\frac{1}{2\xi}(\partial_\mu P^\mu + \xi\partial_y P^y)^2 - \frac{1}{\xi}|\partial_\mu W^{+,\mu} + \xi[\partial_y W^{+,y} - iM_W h^+]|^2 - \frac{1}{2\xi}(\partial_\mu Z^\mu + \xi[\partial_y Z^y - M_Z\chi^0])^2 \tag{A3}$$

to remove the mixing between gauge bosons and Higgs. Therefore, the Goldstone bosons and the physical scalars can be easily identified:

$$G_n^\gamma = P_n^y, \qquad M_\gamma^{(n)} = n/R, \tag{A4}$$

$$G_n^0 = [c_n^Z Z_n^y - s_n^Z \chi_n^0], \qquad S_n^0 = [s_n^Z Z_n^y + c_n^Z \chi_n^0], \qquad M_Z^{(n)} = \sqrt{n^2/R^2 + M_Z^2}, \qquad s_n^Z = M_Z/M_Z^{(n)}, \tag{A5}$$

$$G_n^\pm = [c_n^W W_n^{y\pm} \mp i s_n^W h_n^\pm], \quad H_n^\pm = [s_n^W W_n^{y\pm} \pm i c_n^W h_n^\pm], \quad M_W^{(n)} = \sqrt{n^2/R^2 + M_W^2}, \quad s_n^Z = M_W/M_W^{(n)}, \tag{A6}$$

$$H_n^0 = \phi_n^0, \qquad M_H^{(n)} = \sqrt{n^2/R^2 + M_\phi^2}, \tag{A7}$$

where G^0 and $G^\pm$ are the KK Goldstone bosons, S^0 is the physical KK pseudoscalar, and H^0, $H^\pm$ are the physical KK scalars. The usual R_ξ gauge can be extended to the 5D S_1/Z_2 model with little modification, like $M_W \Rightarrow M_W^{(n)}$ and so on.

The Goldstone bosons are mainly constituted by the fifth gauge components with a small fraction of KK Higgs bosons mixed interaction. On the other hand the Goldstone bosons couple to brane fermions through their Higgs components. In contrast the physical scalars are mainly composed of KK Higgs plus a small amount of the fifth components of gauge fields.

This scheme can also be applied to the models built on the $S_1/(Z_2 \times Z_2')$ orbifold with little modification.

APPENDIX B: GAUGE FIXING FOR THE ORBIFOLD MODELS WITH MORE THAN ONE SCALAR

The method can be easily extended to the cases with multi scalars. Taking a 5D two Higgs doublets Model (2HDM) as an example, with VEVs $\langle\phi_1\rangle = v_1$, $\langle\phi_2\rangle = v_2$ and $\tan\beta = v_2/v_1$, the physical charged scalars and pseudoscalars are

$$H^\pm = \sin\beta\phi_1^\pm - \cos\beta\phi_2^\pm, \qquad A^0 = \sin\beta Im\phi_1^0 - \cos\beta Im\phi_2^0, \tag{B1}$$

just like the usual 4D 2HDM. The only difference is that the orthogonal linear combinations $a^0 = \cos\beta Im\phi_1^0 + \sin\beta Im\phi_2^0$ and $g^\pm = \cos\beta\phi_1^\pm + \sin\beta\phi_2^\pm$ will mix with the fifth components of gauge fields to form the real Goldstone bosons:

$$G_n^0 = \cos\theta_n^0 V_n^0 - \sin\theta_n^0 a_n^0, \qquad \sin\theta_n^0 = M_0/\sqrt{M_0^2 + n^2/R^2}, \tag{B2}$$

$$G_n^\pm = \cos\theta_n^\pm V_n^\pm \mp i\sin\theta_n^\pm g_n^\pm, \qquad \sin\theta_n^\pm = M_V/\sqrt{M_V^2 + n^2/R^2}. \tag{B3}$$

[1] Super-Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1562 (1998).
[2] SNO Collaboration, Q. R. Ahmad *et al.*, Phys. Rev. Lett. **89**, 011301 (2002).
[3] KamLAND Collaboration, K. Eguchi *et al.*, Phys. Rev. Lett. **90**, 021802 (2003).
[4] C. L. Bennett *et al.*, Astrophys. J. Suppl. Ser. **148**, 1 (2003).
[5] A. Masiero, S. K. Vempati, and O. Vives, Nucl. Phys. B **649**, 189 (2003); T. Blazek and S. F. King, Nucl. Phys. B **662**, 359 (2004).
[6] W. F. Chang and J. N. Ng, hep-ph/0411201.
[7] C. H. Chang, W. F. Chang, and J. N. Ng, Phys. Lett. B **558**, 92 (2003).
[8] W. F. Chang and J. N. Ng, J. High Energy Phys. 10 (2003) 036.
[9] J. N. Ng, J. Korean Phys. Soc. **45**, S341 (2004).
[10] Y. Kuno and Y. Okada, Rev. Mod. Phys. **73**, 151 (2001) and references therein.
[11] S. T. Petcov, Phys. Lett. B **68**, 365 (1977).
[12] K. Huitu, J. Maalampi, M. Raidal, and A. Santamaria, Phys. Lett. B **430**, 355 (1998); T. S. Kosmas, S. Kovalenko, and I. Schmidt, Phys. Lett. B **511**, 203 (2001).
[13] G. Feinberg and S. Weinberg, Phys. Rev. Lett. **3**, 527 (1959).
[14] R. Kitano, M. Koike, and Y. Okada, Phys. Rev. D **66**, 096002 (2002).
[15] H. C. Chiang, E. Oset, T. S. Kosmas, A. Faessler, and J. D. Vergados, Nucl. Phys. A **559**, 526 (1993).
[16] S. Weinberg, Phys. Rev. D **5**, 1962 (1975).
[17] L. J. Hall and Y. Normura, Phys. Lett. B **532**, 111 (2002); T. Li and W. Liao, Phys. Lett. B **545**, 147 (2002); S. Dimopoulos, D. E. Kaplan, and N. Weiner, Phys. Lett. B **534**, 124 (2002); I. Antoniadis and K. Benakli, Phys. Lett. B **326**, 69 (1994).
[18] A. Zee, Phys. Lett. B **161**, 141 (1985).
[19] Particle Data Group, Phys. Lett. B **592**, 1 (2004).

[20] BABAR Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **92**, 121801 (2004).
[21] S. Glashow and S. Weinberg, Phys. Rev. D **15**, 1958 (1977).
[22] N. Arkani-Hamed and M. Schmaltz, Phys. Rev. D **61**, 033005 (2000); N. Arkani-Hamed, Y. Grossman, and M. Schmaltz, Phys. Rev. D **61**, 115004 (2000).
[23] E. A. Mirabelli and M. Schmaltz, Phys. Rev. D **61**, 113011 (2000).
[24] G. C. Branco, A. de Gouvea, and M. N. Rebelo, Phys. Lett. B **506**, 115 (2001).
[25] W. F. Chang and J. N. Ng, J. High Energy Phys. 12 (2002) 077.
[26] W. F. Chang, I. L. Ho, and J. N. Ng, Phys. Rev. D **66**, 076004 (2002).
[27] MECO Collaboration, J. L. Popp, Nucl. Instrum. Methods Phys. Res., Sect. A **472**, 354 (2000).
[28] D. M. Ghilencea, S. Groot Nibbelink, and H. P. Nilles, Nucl. Phys. B **619**, 385 (2001); A. Muck, A. Pilaftsis, and R. Ruckl, Phys. Rev. D **65**, 085037 (2002).
[29] For example, K. R. Dienes, E. Dudas, and T. Gherghetta, Nucl. Phys. B **537**, 47 (1999); J. Papavassiliou and A. Santamaria, Phys. Rev. D **63**, 125014 (2001).
[30] A. De Gouvea, G. F. Giudice, A. Strumia, and K. Tobe, Nucl. Phys. B **623**, 395 (2002).